Two field Mimetic gravity

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Mimetic gravity as dark matter model
Chamseddine & Mukhanov 2013

- Consider a physical metric $g_{\mu\nu}$ to be a function of a scalar field ϕ and an auxiliary metric $\tilde{g}_{\mu\nu}$,

$$g_{\mu\nu} = \left(\tilde{g}^{\alpha\beta}\partial_\alpha\phi\partial_\beta\phi\right)\tilde{g}_{\mu\nu} \equiv \tilde{X}\,\tilde{g}_{\mu\nu}.$$

The metric $g_{\mu\nu}$ is *invariant under the conformal transformation* of the auxiliary metric. Note there exist a *constraint equation*

$$g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi = 1.$$

- The action for original mimetic gravity is

$$S = \int d^4x\,\sqrt{-g\left(\tilde{g},\phi\right)}\left[\frac{1}{2}R\left(g\left(\tilde{g},\phi\right)\right) + \mathcal{L}_m\left(g(\tilde{g},\phi)\,,...\right)\right].$$

which is obviously *conformal invariant.*

Mimetic gravity in the Lagrange multiplier formalism

$$S=\int d^4x\sqrt{-g}\left[\frac{1}{2}R\left(g_{\mu\nu}\right)+\mathcal{L}_m\left(g_{\mu\nu},...\right)+\frac{\lambda}{2}\left(g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi-1\right)\right].$$

Equation of motion

$$\begin{aligned} G_{\mu\nu}+T^{(m)}_{\mu\nu}+\lambda\partial_\mu\phi\partial_\nu\phi &= 0, \\ g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi &= 1. \end{aligned}$$

Einstein equation obtains an extra DM term, and the second equation is the mimetic constraint.

Non-invertible disformal transformation

- Note that the transformation $g_{\mu\nu} = \left(\tilde{g}^{\alpha\beta}\partial_\alpha\phi\partial_\beta\phi\right)\tilde{g}_{\mu\nu}$ is *non-invertble*, that's why we have a mimetic constraint.
- Start with more general disformal tranformation

$$g_{\mu\nu} = A(\phi,\tilde{X})\tilde{g}_{\mu\nu} + B(\phi,\tilde{X})\phi_\mu\phi_\nu.$$

The variation of $g_{\mu\nu}$ due to $\tilde{g}_{\mu\nu}$ is

$$\delta g_{\mu\nu} = A\delta\tilde{g}_{\mu\nu} + \left(\tilde{g}_{\mu\nu}A_{\tilde{X}} + \phi_\mu\phi_\nu B_{\tilde{X}}\right)\delta\tilde{g}^{\alpha\beta}\phi_\alpha\phi_\beta.$$

The condition for non-invertible disformal transformation

$$\det\left|\frac{\partial g_{\mu\nu}}{\partial\tilde{g}_{\alpha\beta}}\right| = 0$$

gives us

$$A - \tilde{X}A_{\tilde{X}} - \tilde{X}^2B_{\tilde{X}} = 0 \Rightarrow \frac{A}{\tilde{X}} + B = \frac{1}{f(\phi)}$$

Generalized Mimetic gravity

We obtain the general non-invertible disformal tranformation

$$g_{\mu\nu} = A(\phi, \tilde{X})\tilde{g}_{\mu\nu} + B(\phi, \tilde{X})\phi_\mu\phi_\nu.$$

where

$$\frac{A}{\tilde{X}} + B = \frac{1}{f(\phi)}.$$

The mimetic gravity can be generalized as

$$S = \int d^4x \sqrt{-g\left(\tilde{g}, \phi\right)} \left[\frac{1}{2} R\left(g\left(\tilde{g}, \phi\right)\right) + \mathcal{L}_m\right],$$

and the equivalent lagrangian multiplier formalism is

$$S = \int \mathrm{d}^4x \sqrt{-g} \left[\frac{1}{2} R + \mathcal{L}_m + \frac{\lambda}{2}\left(g^{\mu\nu}\phi_\mu\phi_\nu - b(\phi)\right)\right].$$

Several issues

- Without matter, we have the perturbation equation $\dot{\zeta} = 0$. But in the presence of extra matter field

$$S = \int d^4x\sqrt{-g}\left[\frac{R}{2} + \frac{\lambda}{2}\left(g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - 1\right) - V\left(\phi\right) + \mathcal{L}_m\left(\psi, Y\right)\right]$$

  Where $Y = g^{\mu\nu}\psi_\mu\psi_\nu$, the reduced quadratic action contains the term $\ddot{\zeta}^2$ and ghost exists .

- To obtain mimetic inflation, we usually introduce the higher derivative terms

$$S = \int d^4x\sqrt{-g}\left[\frac{1}{2}R + \frac{\lambda}{2}\left(g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - 1\right) - V\left(\phi\right) + \frac{\gamma}{2}\left(\Box\phi\right)^2\right].$$

  However, there exists gradient instability or ghost instability:

$$S_\zeta^{(2)} = \int d^4xa^3\left(-\frac{2-3\gamma}{\gamma}\dot{\zeta}^2 + \frac{(\partial_i\zeta)^2}{a^2}\right).$$

Non-invertible two field conformal transformation

Consider the two-field conformal transformation

$$g_{\mu\nu} = A\,\tilde{g}_{\mu\nu}$$

where $A = A(\phi, \psi, \tilde{X}, \tilde{Y}, \tilde{Z}) = A(\phi, \psi, \phi_\mu, \psi_\mu, \tilde{g}^{\mu\nu})$ and

$$\tilde{X} = \tilde{g}^{\mu\nu}\phi_\mu\phi_\mu,\ \ \tilde{Y} = \tilde{g}^{\mu\nu}\psi_\mu\psi_\mu,\ \ \tilde{Z} = \tilde{g}^{\mu\nu}\phi_\mu\psi_\mu.$$

The non-invertible condition is (Hassan Firouzjahi et al. 2018)

$$A = \tilde{X}A_{\tilde{X}} + \tilde{Y}A_{\tilde{Y}} + \tilde{Z}A_{\tilde{Z}}\ .$$

Thus

$$A(\phi, \psi, c\tilde{X}, c\tilde{Y}, c\tilde{Z}) = c\,A(\phi, \psi, \tilde{X}, \tilde{Y}, \tilde{Z}).$$

or equivalently

$$A(\phi, \psi, \phi_\mu, \psi_\mu, c\tilde{g}^{\mu\nu}) = c\,A(\phi, \psi, \phi_\mu, \psi_\mu, \tilde{g}^{\mu\nu})$$

which means A is a linear function with respect to $\tilde{g}^{\mu\nu}$.

Two field mimetic gravity

Conformal invariant and mimetic constraint

- The metric $g_{\mu\nu}$ is invariant under the conformal transformation of the auxiliary metric $\tilde{g}_{\mu\nu}$.
- From the Non-invertible two field conformal transformation, one can derive the corresponding mimetic constraint

$$A(\phi, \psi, X, Y, Z) = 1.$$

Then we get the action of two field mimetic gravity

$$S = \int d^4x \sqrt{-g\left(\tilde{g}, \phi, \psi\right)} \left[\frac{1}{2} R\left(g\left(\tilde{g}, \phi, \psi\right)\ \right) + \mathcal{L}_m\left(g(\tilde{g}, \phi, \psi)\,, \ldots\right)\right]$$

and the Lagrange multiplier form

$$S = \int \mathrm{d}^4x\sqrt{-g}\left[\frac{R}{2} + \mathcal{L}_m + \frac{\lambda}{2}\left(A(\phi, \psi, X, Y, Z) - 1\right)\right].$$

Case 1 [Hassan Firouzjahi et al. 2018]

Non-invertible conformal transformation

$$A = \alpha(\phi, \psi)\tilde{X} + \beta(\phi, \psi)\tilde{Y} + \gamma(\phi, \psi)\tilde{Z},$$

and the corresponding mimetic constraint

$$\alpha(\phi, \psi)X + \beta(\phi, \psi)Y + \gamma(\phi, \psi)Z = 1.$$

If imposed the shift symmetry, one can obtain the special case

$$X \pm Y = 1.$$

The action for this case is

$$S = \int d^4x\sqrt{-g}\left[\frac{1}{2}R + \lambda\left(g^{\mu\nu}\phi_{,\mu}\phi_{,\nu} \pm g^{\mu\nu}\psi_{,\mu}\psi_{,\nu} - 1\right)\right]$$

Decompose the perturbations into the adiabatic and entropy modes

$$\delta\sigma = (\cos\theta)\delta\phi + (\sin\theta)\delta\psi,$$
$$\delta s = -(\sin\theta)\delta\phi + (\cos\theta)\delta\psi,$$

where

$$\cos\theta \equiv \frac{\dot{\phi}}{\sqrt{\dot{\phi}^2 + \dot{\psi}^2}}, \quad \sin\theta \equiv \frac{\dot{\psi}}{\sqrt{\dot{\phi}^2 + \dot{\psi}^2}}.$$

Define the curvature perturbation as

$$\zeta \equiv \psi + \frac{H}{\dot{\sigma}}\delta\sigma.$$

Perturbation analysis

- The curvature perturbation is just the same as one field before. Without matter term, $\dot{\zeta} = 0$. In the presence of extra matter field, ghost still exist.

- For the entropy mode, the quadratic action is

$$\mathcal{L}^{(2)}_{\delta s} = \pm(\frac{3}{2}a^3H^2\delta\dot{s}^2 - \frac{3}{2}aH^2k^2\delta s^2) \ .$$

  Whether entropy mode is a ghost or not is up to the sign of two field mimetic constraint !

Case 2

- For non-invertible conformal transformation $A = \frac{\tilde{X}^2}{\tilde{Y}}$, and the corresponding mimetic constraint $X^2 - Y = 0$. The action for this case is
$$S = \int d^4x\sqrt{-g}\left[\frac{1}{2}R + \lambda\left(X^2 - Y\right)\right].$$
- For $A = \sqrt{\tilde{X}\tilde{Y}}$, the corresponding mimetic constraint $XY = 1$. The action is
$$S = \int d^4x\sqrt{-g}\left[\frac{1}{2}R + \lambda\left(XY - 1\right)\right].$$

Case 3

If we take non-invertible transformation

$$A = \tilde{X}/J(\phi, \psi),$$

the mimetic constraint will be

$$\psi = f(\phi, X),$$

where f is the implicit inverse function of J. The action for this case becomes

$$S = \int \mathrm{d}^4x\sqrt{-g}\left[\frac{R}{2} + \lambda(\psi - f(\phi, X))\right].$$

This will go back to GR.

To make it more interesting, we slight modify it (just like we did in one-field mimetic gravity)

- Action I:

$$S = \int d^4x\sqrt{-g}\left[\frac{R}{2} + \lambda(\psi - f(\phi, X)) + h(\phi, \psi, X)\right].$$

By using the two-field mimetic constraint, we have

$$S = \int d^4x\sqrt{-g}\left[\frac{R}{2} + K(\phi, X)\right]$$

where $K(\phi, X) = h(\phi, f(\phi, X), X)$. K-essence gravity !

- Action II:

$$S = \int d^4x\sqrt{-g}\left[\frac{R}{2} + \lambda(\psi - f(\phi, X)) + G_2(\phi, X) + \phi^{\mu}\psi_{\mu}\right]$$

By using the constraint, the action reduces to

$$S = \int d^4x\sqrt{-g}\left[\frac{R}{2} + G_2(\phi, X) + G_3(\phi, X)\Box\phi\right]$$

where $G_3(\phi, X) = -f(\phi, X)$. Galileon gravity !

- Action III:

$$S = \int d^4x\sqrt{-g}\left[\frac{R}{2} + \lambda(\psi - f(\phi, X)) + \psi^\mu\psi_\mu\right]$$

Using the constraint, the action reduces to

$$S = \int d^4x\sqrt{-g}\left[\frac{R}{2} + f(\phi, X)_{,\mu} f(\phi, X)^{,\mu}\right]$$

Higher derivative exist. Ostrogradski ghost !

Summary

- We discussed the relation of non-invertible transformation and mimetic gravity.
- Two-field mimetic gravity may resolve the issues of one-field mimetic gravity.
- Ghost exists and issues remain in some cases.
- We found some interesting cases can go back to the familiar K-essence theory and Galileon gravity which are healthy.

Thank you !